

is question paper contains 4 printed pages]

Roll No. :

No. of Q. Paper : 608 I

Que Paper Code : 32357502

Name of the Course : B.Sc.(Hons.)
Mathematics : DSE - I

Name of the Paper : Mathematical Modelling
& Graph Theory

Semester : V

Time : 3 Hours

Maximum Marks : 75

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) Attempt any **three** parts of each question.

(c) **All** questions are compulsory.

(a) Solve the initial value problem using the Laplace transform :

6

$$x^{(3)} + x'' - 6x' = 0; x(0) = x''(0) = 1$$

b) (i) Find the inverse Laplace transform of :

$$F(s) = \frac{1}{s^2 - 4}$$

2

P.T.O.

(ii) Show that :

$$L\{t \sin kt\} = \frac{2sk}{(s^2 + k^2)^2} .$$

(iii) Find the inverse Laplace trans

$$F(s) = \frac{1}{(s^2 + s - 6)^2} .$$

(c) Find two linearly independent Frobenius series solutions of :

$$6x^2y'' + 7xy' - (x^2 + 2)y = 0 .$$

(d) Use power series to solve the initial value problem :

$$(4x^2 + 16x + 7)y'' - 8y = 0; y(-2) = 1, y'(-2) = 0$$

2. (a) Explain Linear Congruence Method and use it to generate 10 random numbers using $a = 5$, $b = 1$ and $c = 8$. Was there a cycle, so, when did it occur ?

(b) Using Monte Carlo Simulation, design an algorithm to approximate the area under the curve $f(x) = \sqrt{x}$, over the interval $\frac{1}{2}$

(c) Using algebraic analysis : 6

Maximize $5x + 3y$

subject to $x + y \leq 6,$

$3x - y \leq 9,$

$x, y \geq 0.$

(d) Using graphical analysis : 6

Minimize $5x + 7y$

subject to $2x + 3y \geq 6$

$3x - y \leq 15,$

$-x + y \leq 4,$

$2x + 5y \leq 27,$

$x, y \geq 0.$

(a) (i) Is it possible to draw a 3-regular graph with 3 vertices ? 3

(ii) Draw the eleven unlabelled simple graphs with four vertices. 3

(b) (i) Define an Eulerian trail and semi-Eulerian trail. Give **one** example for each. 4

(ii) Draw a simple connected graph with degree sequence $(1, 1, 2, 3, 3, 4, 4, 6).$ 2

(c) Prove that there is no knight's tour on a 3×6 chessboard. 6

(d) Prove that a bipartite graph with odd number of vertices is not Hamiltonian. 6

4. (a) Use the factorization :

$$s^4 + 4a^4 = (s^2 - 2as + 2a^2)(s^2 + 2as + 2a^2)$$

and apply inverse Laplace transform to show that :

$$L^{-1} \left\{ \frac{s^2}{s^4 + 4a^4} \right\} = \frac{1}{2a} (\cosh at \sin at + \sinh at \cos at).$$

- (b) Find the general solutions in power of x of the following differential equation :

$$y'' + xy' + y = 0.$$

- (c) Solve the problem :

$$\text{Maximize } 25x + 30y$$

$$\text{subject to } 20x + 30y \leq 690,$$

$$5x + 4y \leq 120,$$

$$x, y \geq 0.$$

Determine the sensitivity of the optimal solution to change in C_1 using the objective function $C_1x + 30y$.

- (d) Write down a Gray code of 4 - digit binary words.